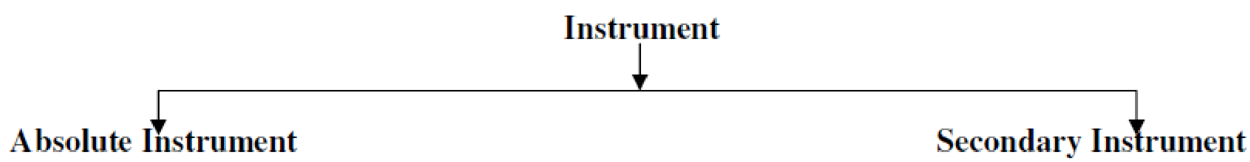
Unit-I

MEASURING INSTRUMENTS

1.1Definition of instruments:

An instrument is a device in which we can determine the magnitude or value of the quantity to be measured. The measuring quantity can be voltage, current, power and energy etc. Generally instruments are classified in to two categories.

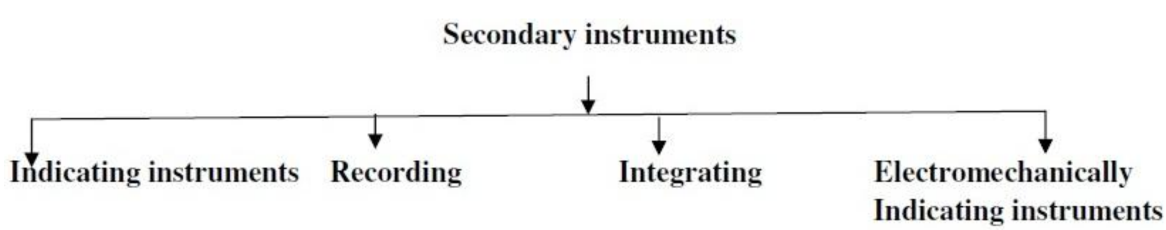


1.2 Absolute instrument:

An absolute instrument determines the magnitude of the quantity to be measured in terms of the instrument parameter. This instrument is really used, because each time the value of the measuring quantities varies. So we have to calculate the magnitude of the measuring quantity, analytically which is time consuming. These types of instruments are suitable for laboratory use.
Example: Tangent galvanometer

1.3 Secondary instrument:

This instrument determines the value of the quantity to be measured directly. Generally these instruments are calibrated by comparing with another standard secondary instrument. Examples of such instruments are voltmeter, ammeter and wattmeter etc. Practically secondary instruments are suitable for measurement.



1.3.1 Indicating instrument:

This instrument uses a dial and pointer to determine the value of measuring quantity. The pointer Indication gives the magnitude of measuring quantity.

1.3.2 Recording instrument:

This type of instruments records the magnitude of the quantity to be measured continuously over a specified period of time.

1.3.3 Integrating instrument:

This type of instrument gives the total amount of the quantity to be measured over a specified Period of time

1.3.4 Electromechanical indicating instrument:

For satisfactory operation electromechanical indicating instrument, three forces are necessary. They are

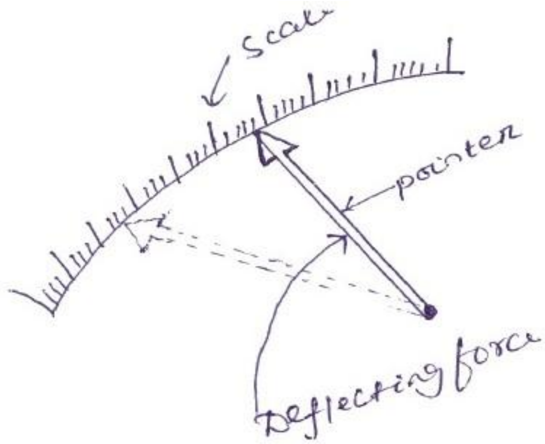
(a) Deflecting force

(b) Controlling force

(c)Damping force

1.4 Deflecting force:

When there is no input signal to the instrument, the pointer will be at its zero position. To deflect the pointer from its zero position, a force is necessary which is known as deflecting force. A system which produces the deflecting force is known as a deflecting system. Generally a deflecting system converts an electrical signal to a mechanical force.



1.5 Controlling force:

To make the measurement indicated by the pointer definite (constant) a force is necessary which will be acting in the opposite direction to the deflecting force. This force is known as controlling force. A system which produces this force is known as a controlled system. When the external signal to be measured by the instrument is removed, the pointer should return back to the zero position. This is possibly due to the controlling force and the pointer will be indicating a steady value when the deflecting torque is equal to controlling torque.

$$T_d = T_c$$

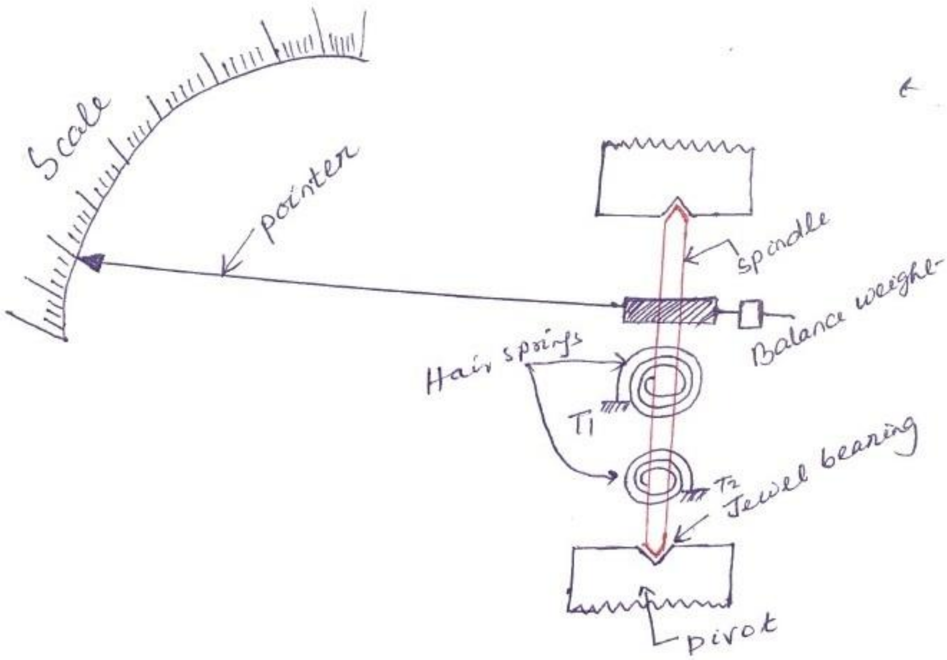
1.5.1 Spring control:

Two springs are attached on either end of spindle (Fig. 1.5).The spindle is placed in jeweled bearing, so that the frictional force between the pivot and spindle will be minimum. Two springs are provided in opposite direction to compensate the temperature error. The spring is made of phosphorous bronze.

$$T_C \propto \theta$$

The deflecting torque produced Td proportional to 'I'. When T_C= Td the pointer will come to a steady position. Therefore

$$\theta \propto I$$



Since, θ and I are directly proportional to the scale of such instrument which uses spring controlled is uniform

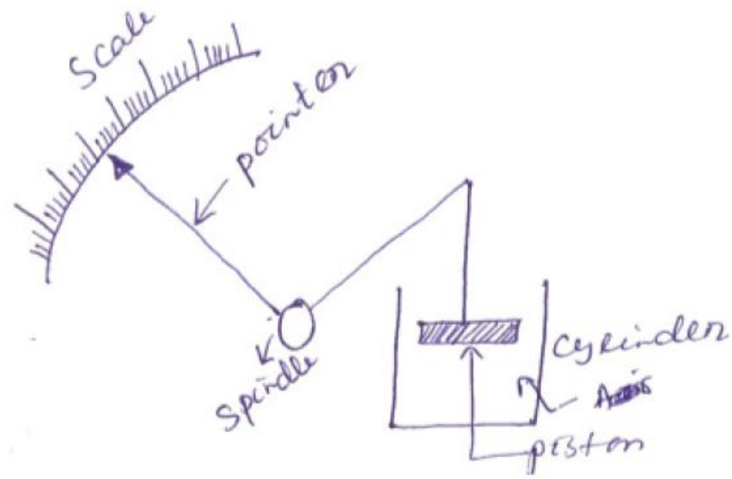
1.6 Damping force:

The deflection torque and controlling torque produced by systems are electro mechanical. Due to inertia produced by this system, the pointer oscillates about it final steady position before coming to rest. The time required to take the measurement is more. To damp out the oscillations quickly, a damping force is necessary. This force is produced by different systems.

(a) Air friction damping
(b) Fluid friction damping
(c) Eddy current damping

1.6.1 Air friction damping:

The piston is mechanically connected to a spindle through the connecting rod (Fig. 1.6). The pointer is fixed to the spindle moves over a calibrated dial. When the pointer oscillates in clockwise direction, the piston goes inside and the cylinder gets compressed. The air pushes the piston upwards and the pointer tends to move in anticlockwise direction



If the pointer oscillates in anticlockwise direction the piston moves away and the pressure of the air inside cylinder gets reduced. The external pressure is more than that of the internal pressure. Therefore the piston moves down wards. The pointer tends to move in clock wise direction.

1.6.2 Eddy current damping:

An aluminum circular disc is fixed to the spindle (Fig. 1.6). This disc is made to move in the magnetic field produced by a permanent magnet.

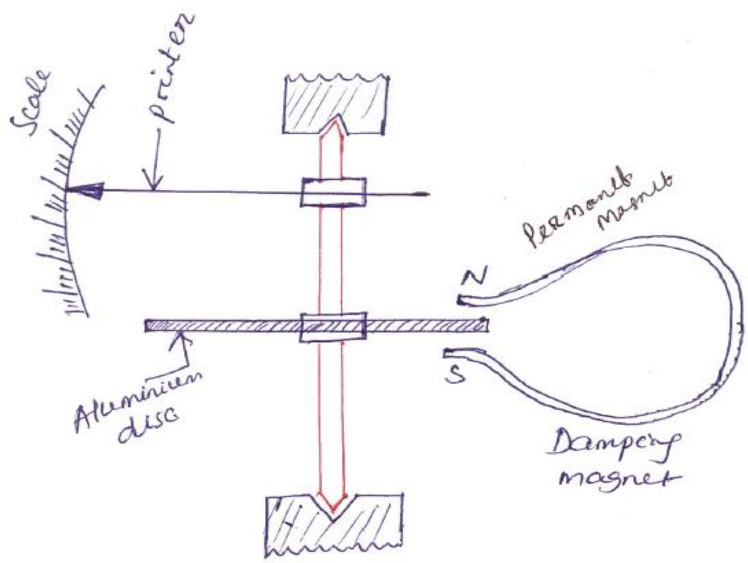


Fig. 1.6 Disc type

When the disc oscillates it cuts the magnetic flux produced by damping magnet. An emf is induced in the circular disc by faradays law. Eddy currents are established in the disc since it has several closed paths. By Lenz's law, the current carrying disc produced a force in a direction opposite to oscillating force. The damping force can be varied by varying the projection of the magnet over the circular disc.

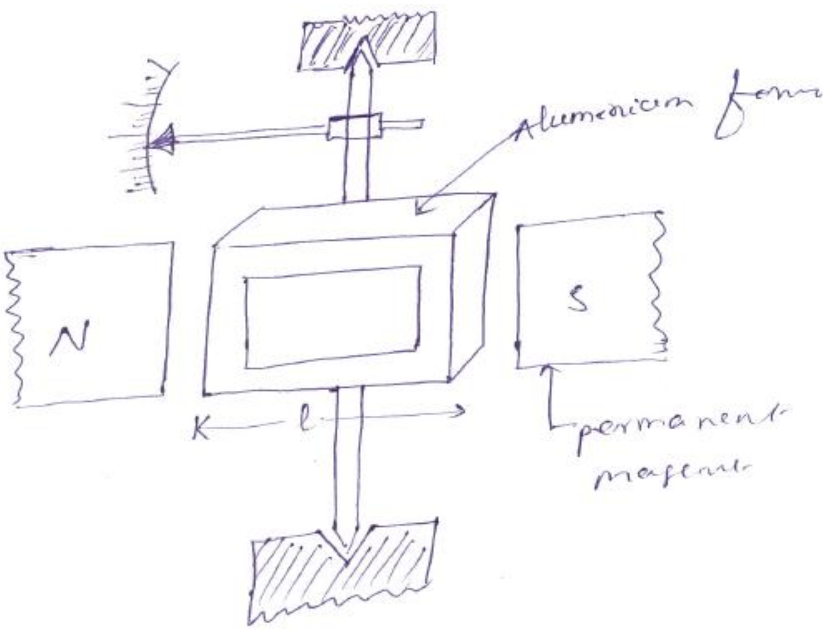


Fig. 1.6 Rectangular type

1.7 Permanent Magnet Moving Coil (PMMC) instrument:

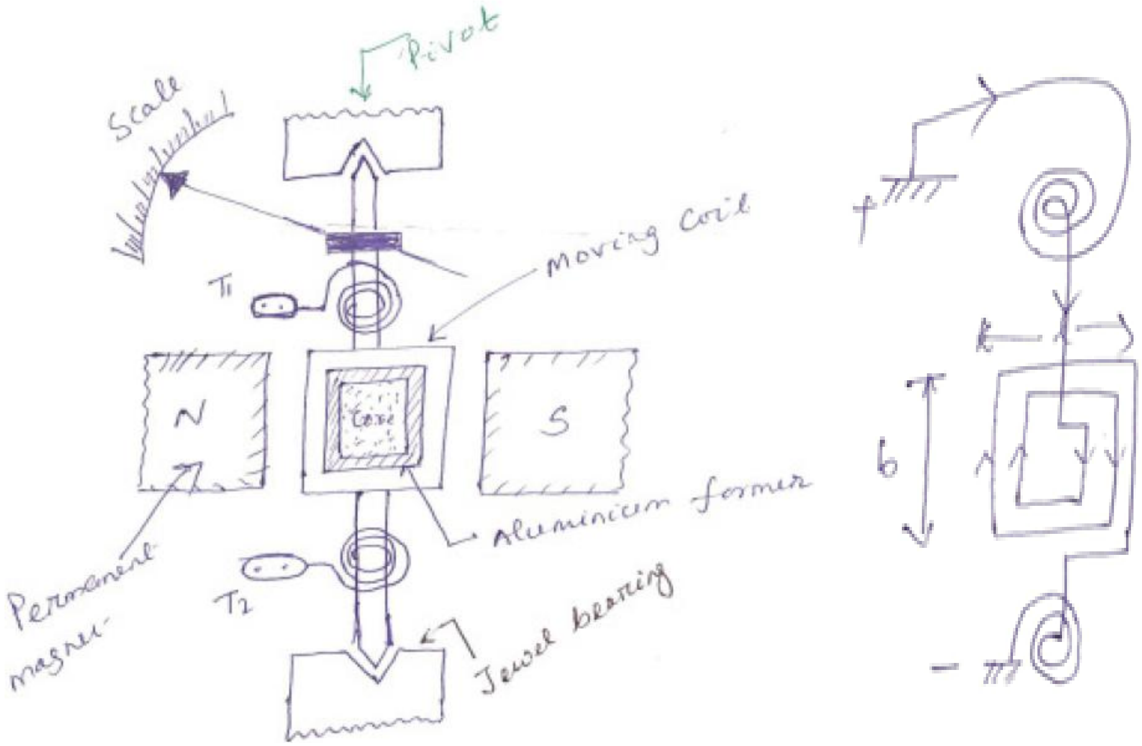
One of the most accurate type of instrument used for D.C. measurements is PMMC instrument.

Construction: A permanent magnet is used in this type instrument. Aluminium former is provided in the cylindrical in between two poles of the permanent magnet (Fig. 1.7). Coils are wound on the aluminium former which is connected with the spindle. This spindle is supported with jewelled bearing. Two springs are attached on either end of the spindle. The terminals of the moving coils are connected to the spring. Therefore the current flows through spring 1, moving coil and spring 2.

Damping: Eddy current damping is used. This is produced by aluminium former.
Control: Spring control is used.

Principle of operation:

When D.C. supply is given to the moving coil, D.C. current flows through it. When the current carrying coil is kept in the magnetic field, it experiences a force. This force produces a torque and the former rotates. The pointer is attached with the spindle. When the former rotates, the pointer moves over the calibrated scale. When the polarity is reversed a torque is produced in the opposite direction. The mechanical stopper does not allow the deflection in the opposite direction. Therefore the polarity should be maintained with PMMC instrument.



If A.C. is supplied, a reversing torque is produced. This cannot produce a continuous deflection. Therefore this instrument cannot be used in A.C.

Torque developed by PMMC:

Let *Td* =deflecting torque

TC = controlling torque

q = angle of deflection

K=spring constant

b=width of the coil

l=height of the coil or length of coil

N=No. of turns

I=current

B=Flux density

A=area of the coil

The force produced in the coil is given by $F = BIL \sin\theta$

When $\theta = 90^o$

For N turns, F = NBIL

Torque produced Td = F* perpendiculardistance

T_d =NBIL* b

= BINA

T_d =BANI

$T_d \alpha I$

Advantages:

- Torque/weight is high
- Power consumption is less
- Scale is uniform
- Damping is very effective
- Since operating field is very strong, the effect of stray field is negligible
- Range of instrument can be extended

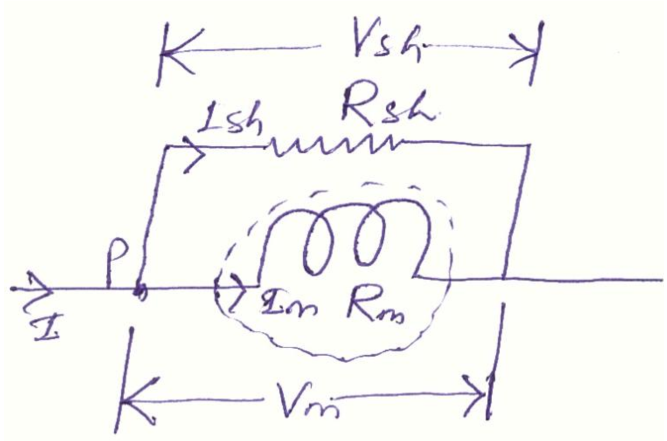
Disadvantages:

- Use only for D.C.
- Cost is high
- Error is produced due to ageing effect of PMMC
- Friction and temperature error are present

1.7.1 Extension of range of PMMC instrument:

Case-I: Shunt:

A low shunt resistance connected in parallel with the ammeter to extent the range of current. Large current can be measured using low current rated ammeter by using a shunt.



Let R_m =Resistance of meter

R_{sh} =Resistance of shunt

I_m = Current through meter

I_{sh}=current through shunt

I = current to be measure

$$\therefore V_m = V_{sh}$$

$$I_m R_m = I_{sh} R_{sh}$$

$$\frac{I_m}{I_{sh}} = \frac{R_{sh}}{R_m}$$

Apply KCL at 'P' $I = I_m + I_{sh}$

Eq^n (1.12) ÷ by I_m

$$\frac{I}{I_m} = 1 + \frac{I_{sh}}{I_m}$$

$$\frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}}$$

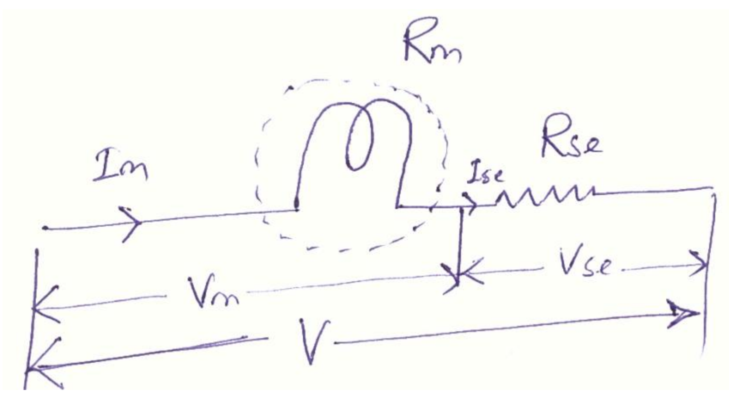
$$\therefore I = I_m\left(1 + \frac{R_m}{R_{sh}}\right)$$

$\left(1 + \frac{R_m}{R_{sh}}\right)$ is called multiplication factor

Shunt resistance is made of manganin. This has least thermo electric emf. The change is resistance, due to change in temperature is negligible

Case (II): Multiplier:

A large resistance is connected in series with voltmeter is called multiplier (Fig. 1.9). A large voltage can be measured using a voltmeter o f small rating with a multiplier.



(Fig. 1.9)

Let R = resistance of meter

R_{se}= resistance of multiplier

V_m = Voltage across meter

V_{se} = Voltage across series resistance

V = voltage to be measured

$$I_m = I_{se}$$

$$\frac{V_m}{R_m} = \frac{V_{se}}{R_{se}}$$

$$\therefore \frac{V_{se}}{V_m} = \frac{R_{se}}{R_m}$$

Apply KVL, $V = V_m + V_{se}$

Eq^n (1.19) $\div V_m$

$$\frac{V}{V_m} = 1 + \frac{V_{se}}{V_m} = \left(1 + \frac{R_{se}}{R_m}\right)$$

$$\therefore V = V_m\left(1 + \frac{R_{se}}{R_m}\right)$$

$$\left(1 + \frac{R_{se}}{R_m}\right) \rightarrow \text{Multiplication factor}$$

1.8 Moving Iron (MI) instruments:

One of the most accurate instruments used for both AC and DC measurement is moving iron instrument. There are two types of moving iron instrument.

- Attraction type
- Repulsion type

1.8.1 Attraction type M.I. instrument

Construction: The moving iron fixed to the spindle is kept near the hollow fixed coil (Fig. 1.10).The pointer and balance weight are attached to the spindle, which is supported with jeweled bearing. Here air friction damping is used.

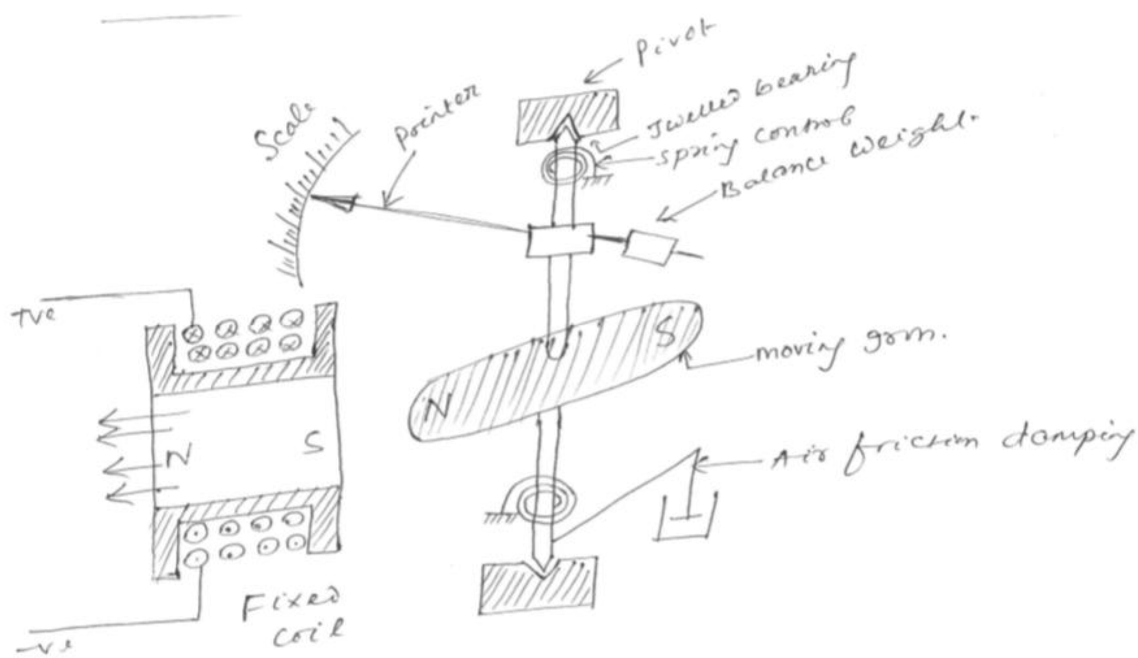
Principle of operation:

The current to be measured is passed through the fixed coil. As the current is flow through the fixed coil, a magnetic field is produced. By magnetic induction the moving iron gets magnetized. The north pole of moving coil is attracted by the south pole of fixed coil. Thus the deflecting force is produced due to force of attraction. Since the moving iron is attached with the spindle, the spindle rotates and the pointer moves over the calibrated scale. But the force of attraction depends on the current flowing through the coil.

Torque developed by M.I

Let ‘θ’ be the deflection corresponding to a current of ‘i’ amp

Let the current increases by di, the corresponding deflection is ‘θ+dθ



(Fig. 1.10)

There is change in inductance since the position of moving iron change w.r.t the fixed electromagnets.

Let the new inductance value be ‘L+dL’. The current change by ‘di’ is dt seconds. Let the emf induced in the coil be ‘e’ volt

It gives the energy is used in to two forms. Part of energy is stored in the inductance. Remaining energy is converted in to mechanical energy which produces deflection

$$e = \frac{d}{dt}(Li) = L\frac{di}{dt} + i\frac{dL}{dt}$$

Multiplying by ‘idt’ in equation

$$e \times idt = L\frac{di}{dt} \times idt + i\frac{dL}{dt} \times idt$$

$$e \times idt = Lidi + i^2 dL$$

(Fig. 1.11)

Change in energy stored=Final energy-initial energy stored

$$= \frac{1}{2}(L + dL)(i + di)^2 - \frac{1}{2}Li^2$$

$$= \frac{1}{2}\{(L + dL)(i^2 + di^2 + 2idi) - Li^2\}$$

$$= \frac{1}{2}\{(L + dL)(i^2 + 2idi) - Li^2\}$$

$$= \frac{1}{2}\{Li^2 + 2Lidi + i^2dL + 2ididL - Li^2\}$$

$$= \frac{1}{2}\{2Lidi + i^2dL\}$$

$$= Lidi + \frac{1}{2}i^2dL$$

Mechanical work to move the pointer by dθ

=Td dθ

By law of conservation of energy, Electrical energy supplied=Increase in stored energy+ mechanical work done

Electrical energy supplied =Increase in stored energy+ mechanical work done

Input energy = Energy stored + Mechanical energy

$$Lidi + i^2 dL = Lidi + \frac{1}{2} i^2 dL + T_d d\theta$$

$$\frac{1}{2} i^2 dL = T_d d\theta$$

$$T_d = \frac{1}{2} i^2 \frac{dL}{d\theta}$$

At steady state condition $T_d = T_C$

$$\frac{1}{2} i^2 \frac{dL}{d\theta} = K\theta$$

$$\theta = \frac{1}{2K} i^2 \frac{dL}{d\theta}$$

$$\theta \propto i^2$$

When the instruments measure AC, $\theta \propto i^2_{rms}$

Scale of the instrument is non uniform.

Advantages:

- MI can be used in AC and DC
- It is cheap
- Supply is given to a fixed coil, not in moving coil.
- Simple construction
- Less friction error.

Disadvantages:

- It suffers from eddy current and hysteresis error.
- Scale is not uniform
- It consumed more power
- Calibration is different for AC and DC operation

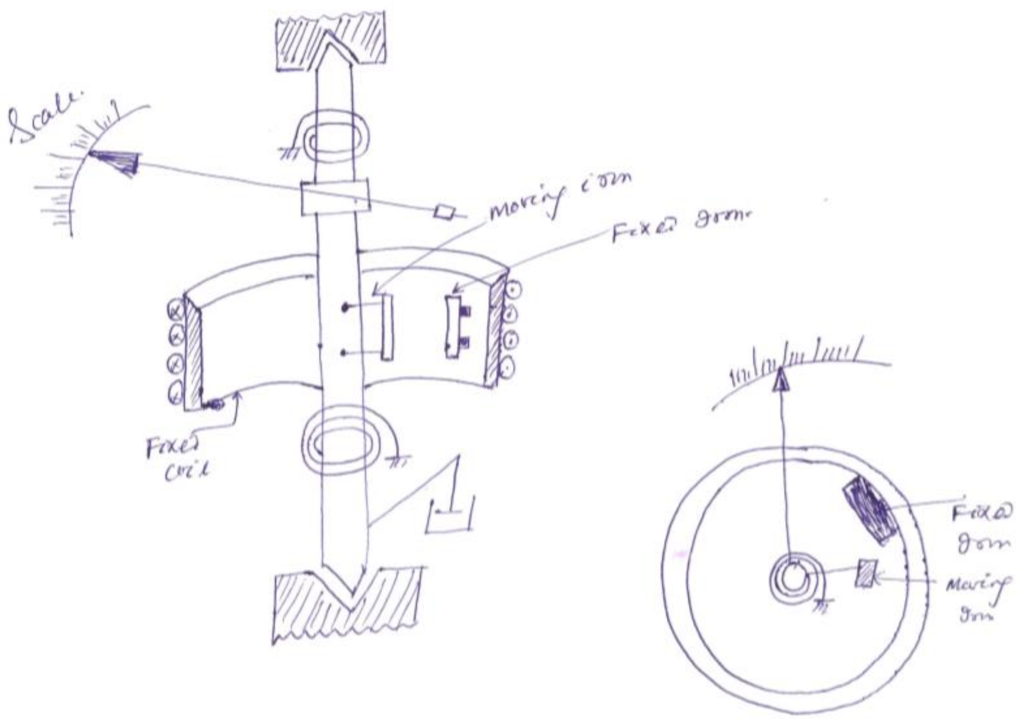
1.8.2 Repulsion type moving iron instrument:

Construction: The repulsion type instrument has a hollow fixed iron attached to it (Fig. 1.12). The moving iron is connected to the spindle. The pointer is also attached to the spindle in supported with jeweled bearing.

Principle of operation: When the current flows through the coil, a magnetic field is produced by it. So both fixed iron and moving iron are magnetized with the same polarity, since they are kept in the same magnetic field. Similar poles of fixed and moving iron get repelled. Thus the deflecting torque is produced due to magnetic repulsion. Since moving iron is attached to spindle, the spindle will move. So that pointer moves over the calibrated scale.

Damping: Air friction damping is used to reduce the oscillation.

Control: Spring control is used.



(Fig. 1.12)

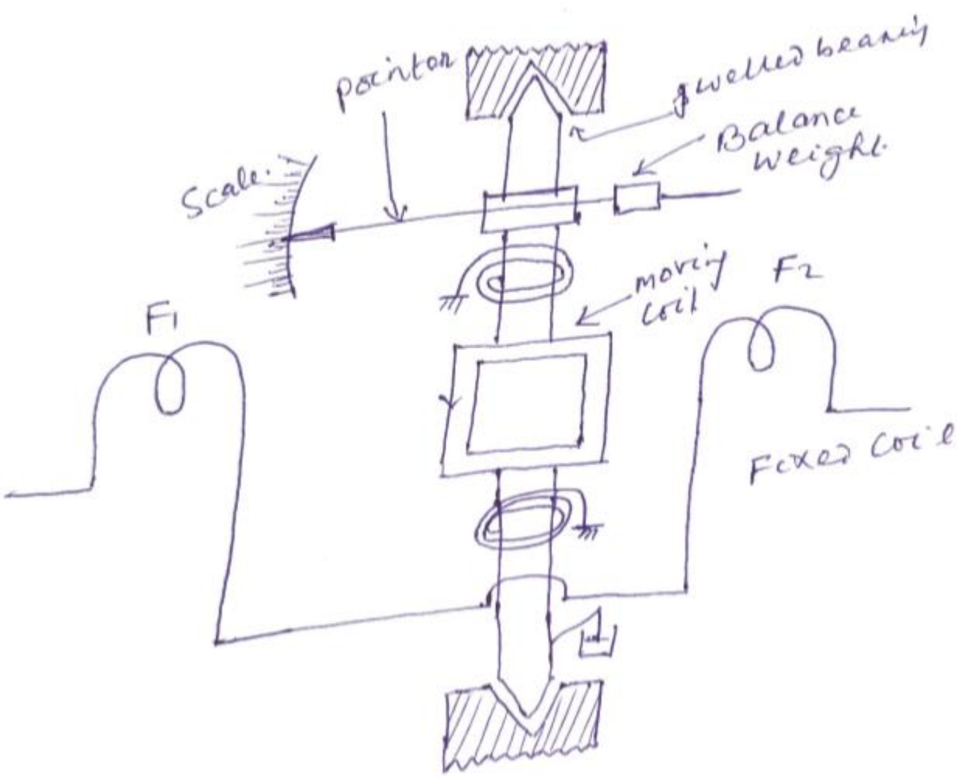
1.9 Dynamometer (or) Electromagnetic moving coil instrument (EMMC):

This instrument can be used for the measurement of voltage, current and power. The difference between the PMMC and dynamometer type instrument is that the permanent magnet is replaced by an electromagnet.

Construction: A fixed coil is divided in to two equal half. The moving coil is placed between the two half of the fixed coil. Both the fixed and moving coils are air cored. So that the hysteresis Effect will be zero. The pointer is attached with the spindle. In a non metallic former the moving Coil is wounded.

Control: Spring control is used.

Damping: Air friction damping is used

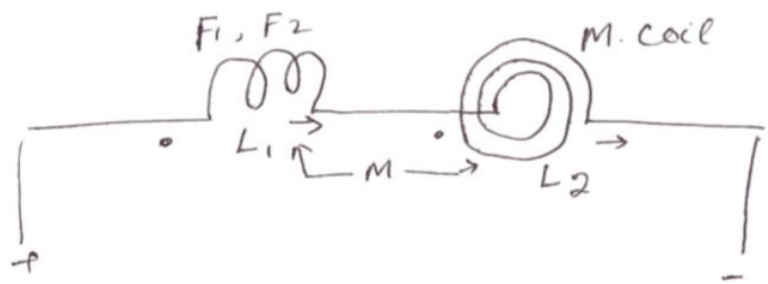


(Fig. 1.13)

Principle of operation:

When the current flows through the fixed coil, it produced a magnetic field, whose flux density is Proportional to the current through the fixed coil. The moving coil is kept in between the fixed coil. When the current passes through the moving coil, a magnetic field is produced by this coil. The magnetic poles are produced in such a way that the torque produced on the moving coil deflects the pointer over the calibrated scale. This instrument works on AC and DC. When AC voltage is applied, alternating current flows through the fixed coil and moving coil. When the current in the fixed coil reverses, the current in the moving coil also reverses. Torque remains in the same direction. Since the current i1 and i2 reverse simultaneously. This is because the fixed and moving coils are either connected in series or parallel.

Torque developed by EMMC:



(Fig. 1.14)

Let

L1=Self inductance of fixed coil

L2= Self inductance of moving coil

M=mutual inductance between fixed coil and moving coil

i1=current through fixed coil

i2=current through moving coil

Total inductance of system

$$L_{total} = L_1 + L_2 + 2M$$

But we know that in case of M.I

$$T_d = \frac{1}{2} i^2 \frac{d(L)}{d\theta}$$

$$T_d = \frac{1}{2} i^2 \frac{d}{d\theta}(L_1 + L_2 + 2M)$$

The value of L_1 and L_2 are independent of 'θ' but 'M' varies with θ

$$T_d = \frac{1}{2} i^2 \times 2 \frac{dM}{d\theta}$$

$$T_d = i^2 \frac{dM}{d\theta}$$

If the coils are not connected in series $i_1 \neq i_2$

$$\therefore T_d = i_1 i_2 \frac{dM}{d\theta}$$

$$T_C = T_d$$

$$\therefore \theta = \frac{i_1 i_2}{K} \frac{dM}{d\theta}$$

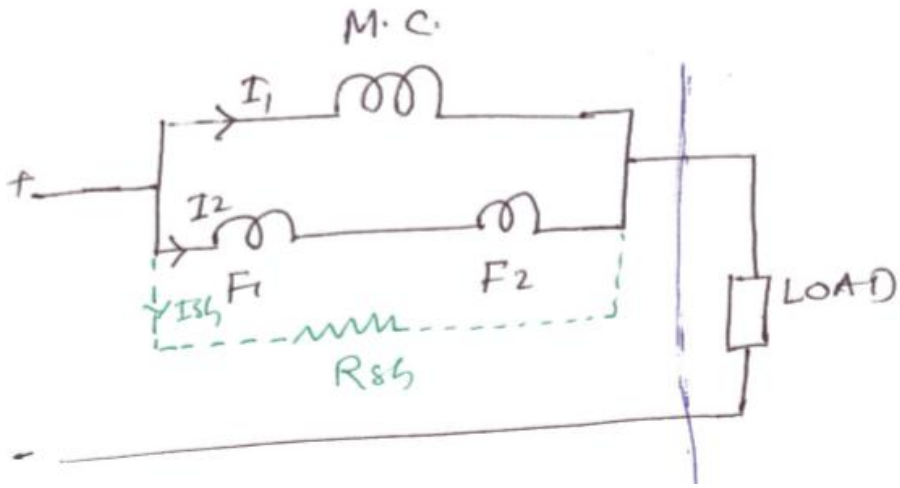
Hence the deflection of pointer is proportional to the current passing through fixed coil and moving coil

1.9.1 Extension of EMMC instrument:

Case-I Ammeter connection

Fixed coil and moving coil are connected in parallel for ammeter connection. The coils are designed such that the resistance of each branch is same. Therefore

$$I_1 = I_2 = I$$



(Fig. 1.15)

To extend the range of current a shunt may be connected in parallel with the meter. The value Rsh is designed such that equal current flows through moving coil and fixed coil

$$\therefore T_d = I_1 I_2 \frac{dM}{d\theta}$$

Or $\therefore T_d = I^2 \frac{dM}{d\theta}$

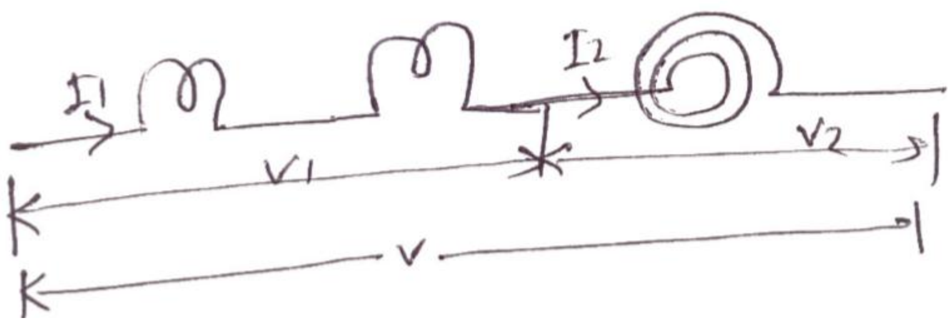
$$T_C = K\theta$$

$$\theta = \frac{I^2}{K}\frac{dM}{d\theta}$$

$\therefore \theta \propto I^2$ (Scale is not uniform)

Case-II Voltmeter connection:

Fixed coil and moving coil are connected in series for voltmeter connection. A multiplier may be Connected in series to extent the range of voltmeter



(Fig. 1.16)

$$I_1 = \frac{V_1}{Z_1}, I_2 = \frac{V_2}{Z_2}$$

$$T_d = \frac{V_1}{Z_1} \times \frac{V_2}{Z_2} \times \frac{dM}{d\theta}$$

$$T_d = \frac{K_1 V}{Z_1} \times \frac{K_2 V}{Z_2} \times \frac{dM}{d\theta}$$

$$T_d = \frac{KV^2}{Z_1 Z_2} \times \frac{dM}{d\theta}$$

$$T_d \propto V^2$$

$\therefore \theta \propto = V^2$ (scale is not uniform)

Case-III: As wattmeter

When the two coils are connected to parallel, the instrument can be used as a wattmeter. Fixed coil is connected in series with the load. Moving coil is connected in parallel with the load. The moving coil is known as voltage coil or pressure coil and fixed coil is known as current coil.

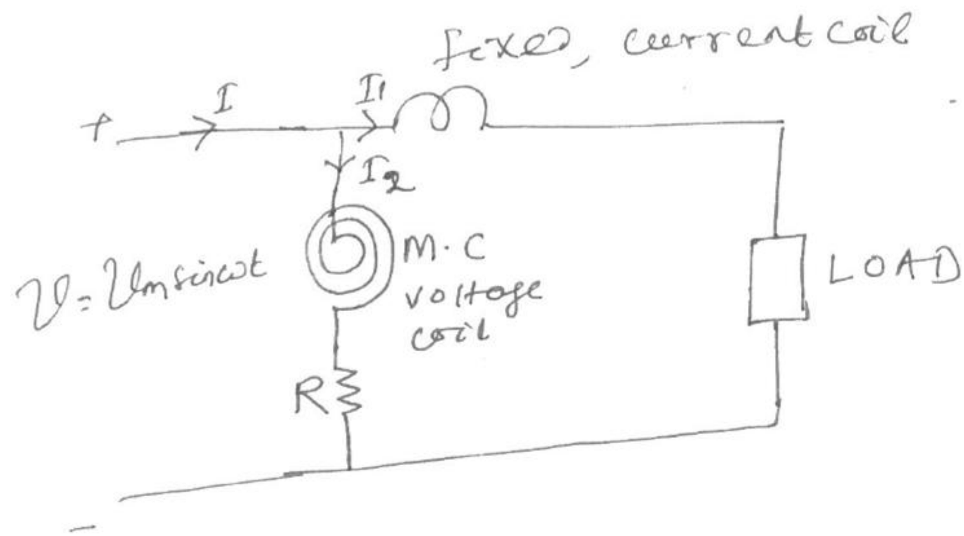


Fig. 1.17

Assume that the supply voltage is sinusoidal. If the impedance of the coil is neglected in comparison with the resistance 'R' The current

$$I_2 = \frac{v_m \sin wt}{R}$$

Let the phase difference between the currents I_1 and I_2 is ϕ

$$I_1 = I_m \sin(wt - \phi)$$

$$T_d = I_1 I_2 \frac{dM}{d\theta}$$

$$T_d = I_m \sin(wt - \phi) \times \frac{V_m \sin wt}{R} \frac{dM}{d\theta}$$

$$T_d = \frac{1}{R}(I_m V_m \sin wt \sin(wt - \phi)) \frac{dM}{d\theta}$$

$$T_d = \frac{1}{R} I_m V_m \sin wt . \sin(wt - \phi) \frac{dM}{d\theta}$$

The average deflecting torque

$$(T_d)_{avg} = \frac{1}{2\Pi}\int_0^{2\Pi} T_d \times d(wt)$$

$$(T_d)_{avg} = \frac{1}{2\Pi}\int_0^{2\Pi} \frac{1}{R} \times I_m V_m \sin wt.\sin(wt-\phi)\frac{dM}{d\theta} \times d(wt)$$

$$(T_d)_{avg} = \frac{V_m I_m}{2\times 2\Pi} \times \frac{1}{R} \times \frac{dM}{d\theta}\left[\int \{\cos\phi - \cos(2wt-\phi)\}dwt\right]$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\Pi R} \times \frac{dM}{d\theta}\left[\int_0^{2\Pi} \cos\phi.dwt - \int_0^{2\Pi} \cos(2wt-\phi).dwt\right]$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\Pi R} \times \frac{dM}{d\theta}\left[\cos\phi[wt]_0^{2\Pi}\right]$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\Pi R} \times \frac{dM}{d\theta}[\cos\phi(2\Pi - 0)]$$

$$(T_d)_{avg} = \frac{V_m I_m}{2} \times \frac{1}{R} \times \frac{dM}{d\theta} \times \cos\phi$$

$$(T_d)_{avg} = V_{rms} \times I_{rms} \times \cos\phi \times \frac{1}{R} \times \frac{dM}{d\theta}$$

$$(T_d)_{avg} \propto KVI\cos\phi$$

$$T_C \propto \theta$$

$$\theta \propto KVI\cos\phi$$

$$\theta \propto VI\cos\phi$$

Advantages:

- It can be used for voltmeter, ammeter and wattmeter
- Hysteresis error is nil
- Eddy current error is nil
- Damping is effective

- It can be measure correctively and accurately the rms value of the voltage

Disadvantages:

- Scale is not uniform
- Power consumption is high(because of high resistance)
- Cost is more
- Error is produced due to frequency, temperature and stray field.
- Torque/weight is low.(Because field strength is very low)

Errors in PMMC:

- The permanent magnet produced error due to ageing effect. By heat treatment, this error can be eliminated.
- The spring produces error due to ageing effect. By heat treating the spring the error can be eliminated.
- When the temperature changes, the resistance of the coil vary and the spring also produces error in deflection. This error can be minimized by using a spring whose temperature co-efficient is very low.

1.10 Electrostatic instrument:

In multi cellular construction several vans and quadrants are provided. The voltage is to be measured is applied between the vanes and quadrant. The force of attraction between the vanes and quadrant produces a deflecting torque. Controlling torque is produced by spring control. Air Friction damping is used.

The instrument is generally used for measuring medium and high voltage. The voltage is reduced to low value by using capacitor potential divider. The force of attraction is proportional to the square of the voltage

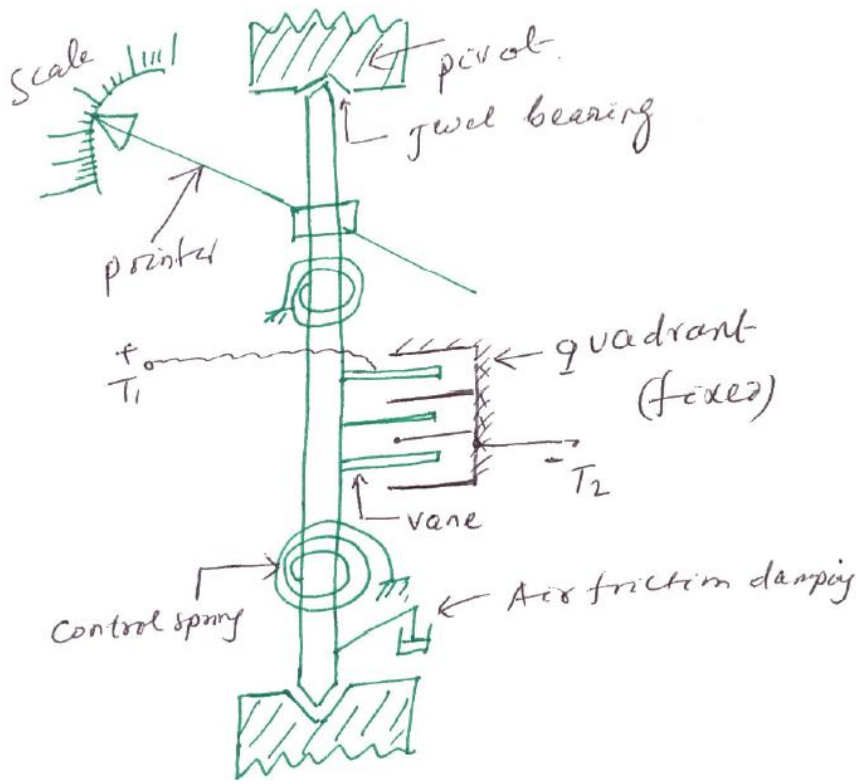


Fig. 1.19

Torque developed by electrostatic instrument:

V=Voltage applied between vane and quadrant

C=capacitance between vane and quadrant

Energy stored = $\frac{1}{2}CV^2$

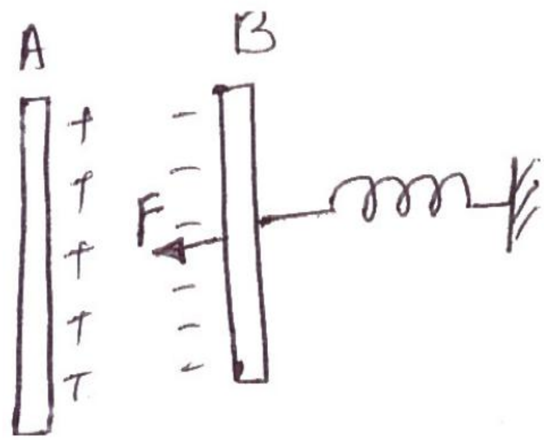
Let ‘θ’ be the deflection corresponding to a voltage V.

Let the voltage increases by dv, the corresponding deflection is’ $\theta + d\theta$ ’

When the voltage is being increased, a capacitive current flows

$$i = \frac{dq}{dt} = \frac{d(CV)}{dt} = \frac{dC}{dt}V + C\frac{dV}{dt}$$

V * dtmultiply on both side of equation



$$Vidt = \frac{dC}{dt}V^2 dt + CV\frac{dV}{dt}dt$$

$$Vidt = V^2 dC + CVdV$$

Change in stored energy= $\frac{1}{2}(C + dC)(V + dV)^2 - \frac{1}{2}CV^2$

$$= \frac{1}{2}\left[(C + dC)V^2 + dV^2 + 2VdV\right] - \frac{1}{2}CV^2$$

$$= \frac{1}{2}\left[CV^2 + CdV^2 + 2CVdV + V^2 dC + dCdV^2 + 2VdVdC\right] - \frac{1}{2}CV^2$$

$$= \frac{1}{2}V^2 dC + CVdV$$

$$V^2 dC + CVdV = \frac{1}{2}V^2 dC + CVdV + F \times rd\theta$$

$$T_d \times d\theta = \frac{1}{2}V^2 dC$$

$$T_d = \frac{1}{2}V^2\left(\frac{dC}{d\theta}\right)$$

At steady state condition, $T_d = T_C$

$$K\theta = \frac{1}{2}V^2\left(\frac{dC}{d\theta}\right)$$

$$\theta = \frac{1}{2K}V^2\left(\frac{dC}{d\theta}\right)$$

Advantages:

- It is used in both AC and DC.
- There is no frequency error.
- There is no hysteresis error.
- There is no stray magnetic field error. Because the instrument works on electrostatic principle.
- It is used for high voltage
- Power consumption is negligible

Disadvantages:

- Scale is not uniform
- Large in size
- Cost is more

1.11Multi range Ammeter:

When the switch is connected to position (1), the supplied current I_1

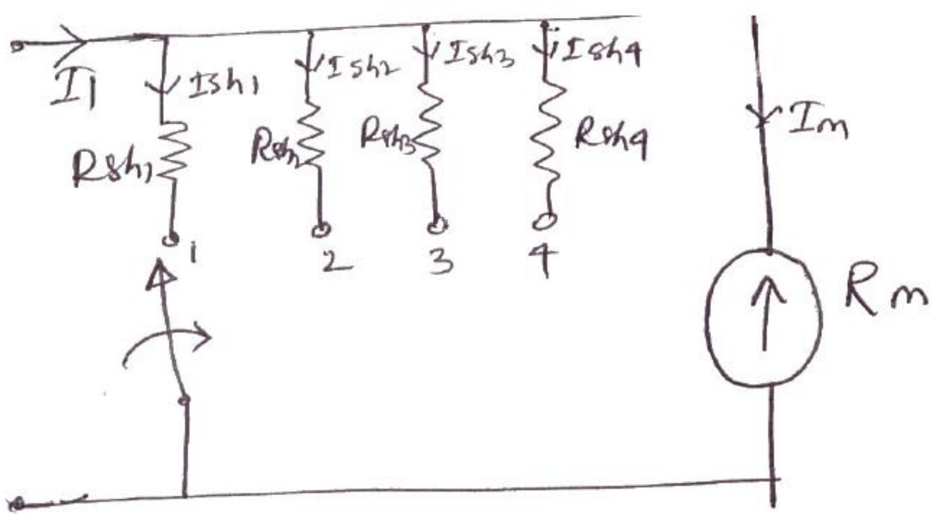


Fig. 1.21

$$I_{sh1}R_{sh1} = I_m R_m$$

$$R_{sh1} = \frac{I_m R_m}{I_{sh1}} = \frac{I_m R_m}{I_1 - I_m}$$

$$R_{sh1} = \frac{R_m}{\frac{I_1}{I_m} - 1}, R_{sh1} = \frac{R_m}{m_1 - 1}, m_1 = \frac{I_1}{I_m} = \text{Multiplying power of shunt}$$

$$R_{sh2} = \frac{R_m}{m_2 - 1}, m_2 = \frac{I_2}{I_m}$$

$$R_{sh3} = \frac{R_m}{m_3 - 1}, m_3 = \frac{I_3}{I_m}$$

$$R_{sh4} = \frac{R_m}{m_4 - 1}, m_4 = \frac{I_4}{I_m}$$

1.15 Ayrton shunt:

$$R_1 = R_{sh1} - R_{sh2}$$

$$R_2 = R_{sh2} - R_{sh3}$$

$$R_3 = R_{sh3} - R_{sh4}$$

$$R_4 = R_{sh4}$$

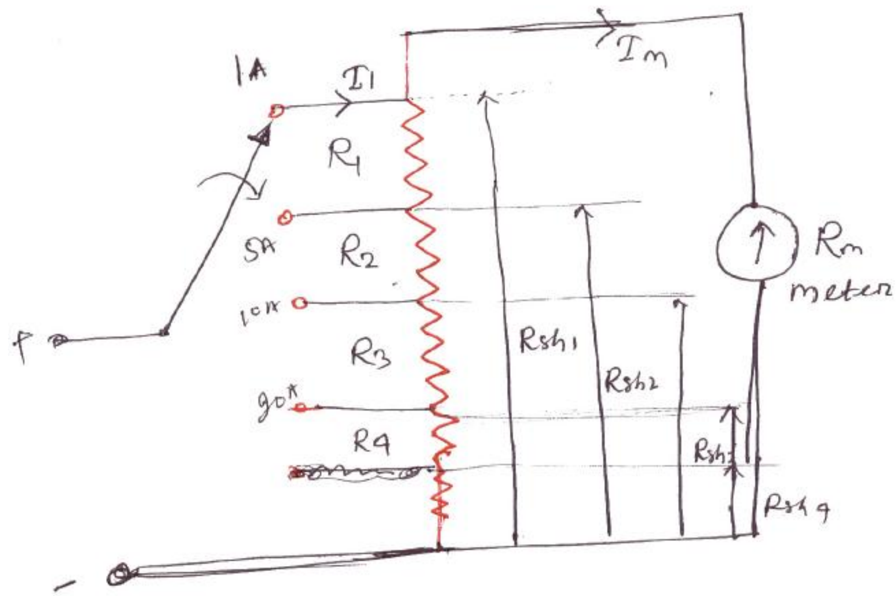
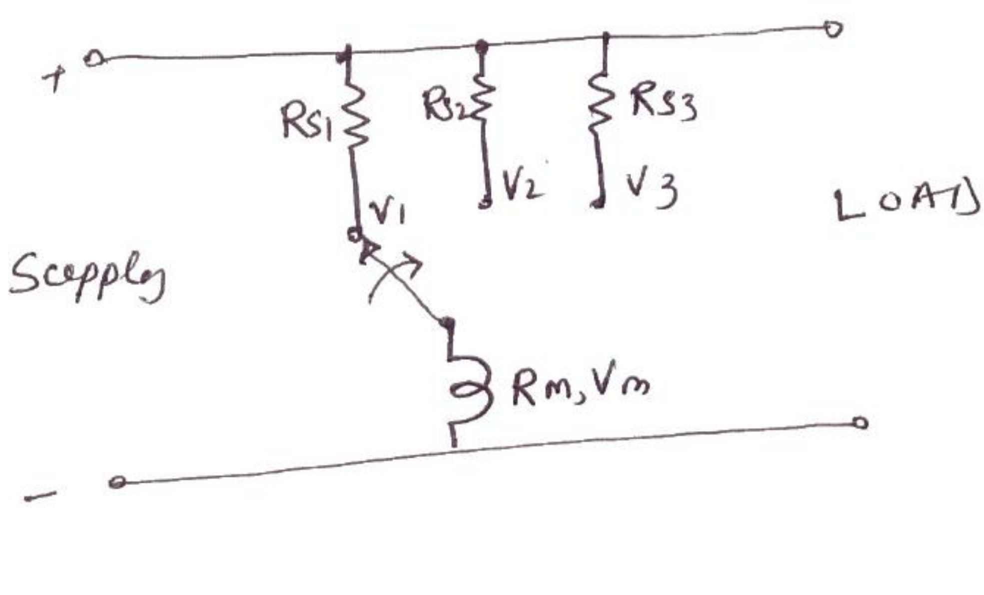


Fig. 1.22

Ayrton shunt is also called universal shunt. Ayrton shunt has more sections of resistance. Taps are brought out from various points of the resistor. The variable points in the o/p can be connected to any position. Various meters require different types of shunts. The Aryton shunt is used in the lab, so that any value of resistance between minimum and maximum specified can be used. It eliminates the possibility of having the meter in the circuit without a shunt.

1.16 Multi range D.C. voltmeter:



$$R_{s1} = R_m(m_1 - 1)$$
$$R_{s2} = R_m(m_2 - 1)$$
$$R_{s3} = R_m(m_3 - 1)$$

$$m_1 = \frac{V_1}{V_m}, m_2 = \frac{V_2}{V_m}, m_3 = \frac{V_3}{V_m}$$

We can obtain different Voltage ranges by connecting different value of multiplier resistor in series with the meter. The number of these resistors is equal to the number of ranges required**.**

Problems:

1. A moving coil instrument gives a full scale deflection of 10mA, when the potential difference across its terminal is 100mV. Calculate

(A) The shunt resistance for a full scale deflection corresponding to 100A

(B) The resistance for full scale reading with 1000V.

(C) Calculate the power dissipation in each case?

Solution:

Data given

$I_m = 10mA$

$V_m = 100mV$

$I = 100A$

$$I = I_m\left(1+\frac{R_m}{R_{sh}}\right)$$

$$100 = 10\times10^{-3}\left(1+\frac{10}{R_{sh}}\right)$$

$R_{sh} = 1.001\times10^{-3}\Omega$

$R_{se} = ??, V = 1000V$

$$R_m = \frac{V_m}{I_m} = \frac{100}{10} = 10\Omega$$

$$V = V_m\left(1+\frac{R_{se}}{R_m}\right)$$

$$1000 = 100\times10^{-3}\left(1+\frac{R_{se}}{10}\right)$$

$\therefore R_{se} = 99.99K\Omega$

2. The inductance of a moving iron instrument is given by

$L = 10 + 5\theta - \theta^2 - \theta^3 \mu H$

Where 'θ' is the deflection in radian from zero position the spring constant is $12\times10^{-6} N-m/rad$. Estimate the deflection for a current of 5A

$$\frac{dL}{d\theta} = (5 - 2\theta)\frac{\mu H}{rad}$$

$$\therefore \theta = \frac{1}{2K} I^2 \left(\frac{dL}{d\theta}\right)$$

$$\therefore \theta = \frac{1}{2} \times \frac{(5)^2}{12 \times 10^{-6}} (5 - 2\theta) \times 10^{-6}$$

$$\therefore \theta = 1.69 rad, 96.8^{\circ}$$

Instrument Transformers:

The d.c circuits when large currents are to be measured, it is usual to use low-range ammeters with suitable shunts. For measuring high voltages, low-range voltmeters are used with high resistances connected in series with them. But it is neither convenient nor practical to use this method with alternating current and voltage instruments. For this purpose, specially constructed accurate-ratio instrument transformers are employed in conjunction with standard low-range a.c. instruments. Their purpose is to reduce the line current or supply voltage to a value small enough to be easily measured with meters of moderates size and capacity. In other words, they are used for extending the range of a.c. ammeters and voltmeters.

Instruments transformers are of two types:

(i) Current transformers (CT) —for measuring large alternating currents.

(ii) potential transformers (VT) —for measuring high alternating voltages

Advantages of using instrument transformers for range extension of a.c. meters are as follows:

(1) The instrument is insulated from the line voltage, hence it can be grounded.

(2) The cost of the instrument (or meter) together with the instrument transformer is less than that of the instrument alone if it were to be insulated for high voltages.

(3) It is possible to achieve standardization instruments and meters at secondary ratings of 100–120 volts and 5 or 1 amperes

(4) If necessary, several instruments can be operated from a single transformer and 5 power consumed in the measuring circuits is low.

In using instrument transformers for current (or voltage) measurements, we must know the ratio of primary current (or voltage) to the secondary current (or voltage). These ratios give us the multiplying factor for finding the primary values from the instrument readings on the secondary side.

However, for energy or power measurements, it is essential to know not only the transformation Ratio but also the phase angle between the primary and secondary currents (or voltages) because it necessitates further correction to the meter reading.

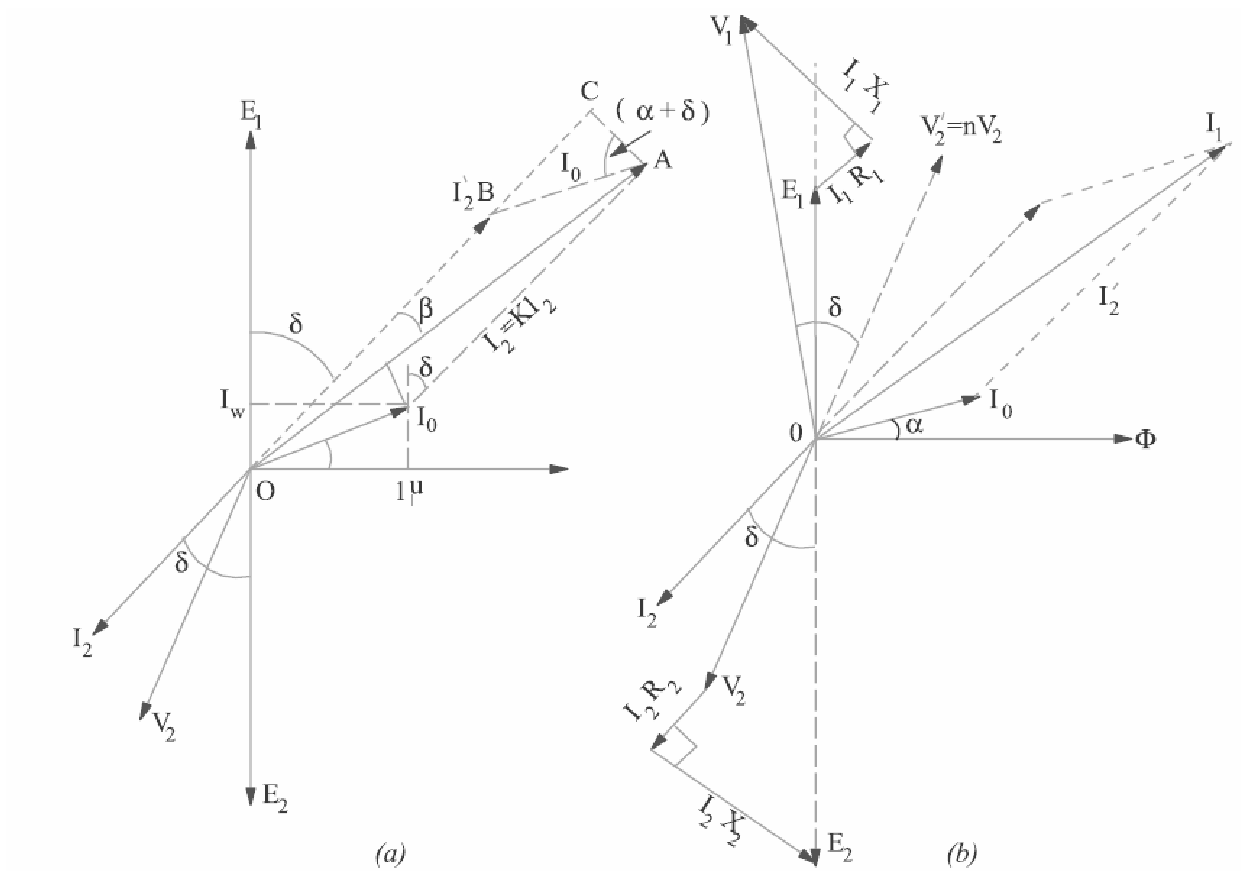
For range extension on a.c. circuits, instrument transformers are more desirable than shunts (for current) and multipliers (for voltage measurements) for the following reasons:

1. Time constant of the shunt must closely match the time constant of the instrument. Hence, a different shunt is needed for each instrument.
2. Range extension is limited by the current-carrying capacity of the shunt *i.e.* up to a few hundred amperes at the most.
3. If current is at high voltage, instrument insulation becomes a very difficult problem.
4. Use of multipliers above 1000 becomes almost impracticable.
5. Insulation of multipliers against leakage current and reduction of their distributed capacitance becomes not only more difficult but expensive above a few thousand volts.

Ratio and Phase-angle Errors:

For satisfactory and accurate performance, it is necessary that the ratio of transformation of the instrument transformer should be constant within close limits. However, in practice, it is found that neither current transformation ratio I_1/I_2 (in the case of current transformers) nor voltage transformation ratio V_1/V_2 (in the case of potential transformers) remains constant. The transformation ratio is found to depend on the exciting current as well as the current and the power factor of the secondary circuit. This fact leads to an error called ratio error of the transformer which depends on the working component of primary.

It is seen from Figure that the phase angle between the primary and secondary currents is not exactly 180° but slightly less than this value. This difference angle β may be found by reversing



Vector $I2$. The angular displacement between $I1$ and $I2$ reversed is called the phase angle error of the current transformer. This angle is reckoned positive if the reversed secondary current ***leads*** the primary current. However, on very low power factors, the phase angle may be negative. Similarly, there is an angle of γ between the primary voltage $V1$ and secondary voltage reversed-this angle represents the phase angle error of a voltage transformer. In either case, the phase error depends on the magnetizing component $I\mu$ of the primary current. It may be noted that ratio error is primarily due to the reason that the ***terminal*** voltage transformation ratio of a transformer is not exactly equal to its turn ratio. The divergence between the two depends on the resistance and reactance of the transformer windings as well as upon the value of the exciting current of the transformer. Accuracy of voltage ratio is of utmost importance in a voltage transformer although phase angle error does not matter if it is to be merely connected *to* ***a voltmeter*.** Phase-angle error becomes important only when voltage transformer supplies the voltage coil of a wattmeter *i.e.* in power measurement. In that case, phase angle error causes the wattmeter to indicate on a wrong power factor.

In the case of current transformers, constancy of current ratio is of paramount importance. Again, Phase angle error is of no significance if the current transformer is merely feeding an ammeter but it assumes importance when feeding the current coil of a wattmeter. While discussing errors, it is worthwhile to define the following terms:

Nominal transformation ratio (kn). It is the ratio of the rated primary to the rated secondary current (or voltage).

$$k_n = \frac{\text{rated primary current } (I_1)}{\text{rated secondary current } (I_2)} \qquad \text{—for } CT$$

$$= \frac{\text{rated primary voltage } (V_1)}{\text{rated secondary voltage } (V_2)} \qquad \text{—for } VT$$

In the case of current transformers, it may be stated either as a fraction such as 500/5 or 100/1 or simply as the number representing the numerator of the reduced fraction *i.e.*100. It is also known as ***marked*** ratio.

Actual transformation ratio (k): The actual transformation ratio or just ratio under any given condition of loading is

$$k = \frac{\text{primary current } (I_1)}{\text{corresponding secondary current } (I_2)}$$

In general, *k* differs from *kn*except in the case of an ideal or perfect transformer when *k* = *kn*for all conditions of loading

Ratio Error (σ): In most measurements it may be assumed that $I_1 = knI_2$ but for very accurate work, it is necessary to correct for the difference between k and kn. It can be done with the help of ratio error which is defined as

$$\sigma = \frac{k_n - k}{k} = \frac{\text{nominal ratio-actual ratio}}{\text{actual ratio}}$$

$$\sigma = \frac{k_n \, . \, I_2 - kI_2}{k \, . \, I_2} = \frac{k_n \, . \, I_2 - I_1}{I_1}$$

Accordingly, ratio error may be defined as the difference between the primary current reading (assuming the nominal ratio) and the true primary current divided by the true primary current

Ratio Correction Factor (R.C.F.): It is given by

$$R.C.F. = \frac{\text{actual ratio}}{\text{nominal ratio}} \quad \frac{k}{k_n}$$

Current Transformer:

A current transformer takes the place of shunt in d.c. measurements and enables heavy alternating current to be measured with the help of a standard 5-A range a.c. ammeter. As shown in Figure below, the current - or series-transformer has a primary winding of one or more turns of thick wire connected in series with the line carrying the current to be measured. The secondary consist of a large number of turns of fine wire and feeds a standard 5-A ammeter or the current coil of a watt-meter or watt-hour-meter (in the below figure)

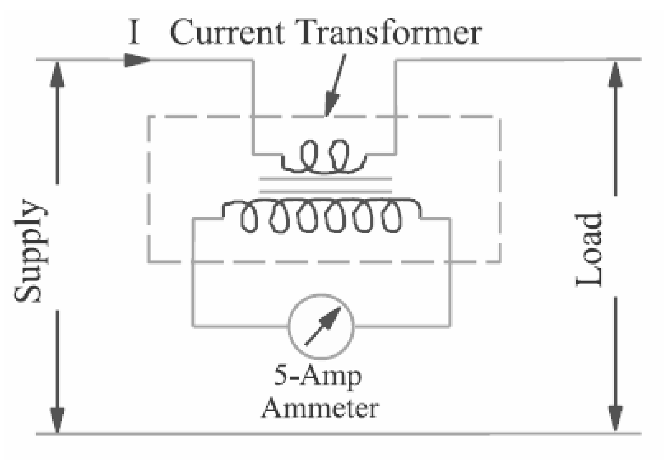


Figure: A

For example, a 1,000/5A current transformer with in single turn primary will have 200 secondary turns. Obviously, it steps down the current in the 200: 1 ratio whereas it steps up the voltage drop across the single-turn primary (an extremely small quantity) in the ratio 1: 200. Hence if we know the current ratio of the transformer and the reading of the a.c. ammeter, the line current can Be calculated.

It is worth noting that ammeter resistances being extremely low, a current transformer operates with its secondary under nearly short-circuit conditions. Should it be necessary to remove the ammeter of the current coils of the wattmeter or a relay, the secondary winding must, first of all, be short-circuited before the instrument is disconnected.

If it is not done then due to the absence of counter ampere-turns of the secondary, the unopposed primary m.m.f. will set up an abnormally high flux in the core which will produce excessive core loss with subsequent heating of and damage of the transformer insulation and a high voltage across the secondary

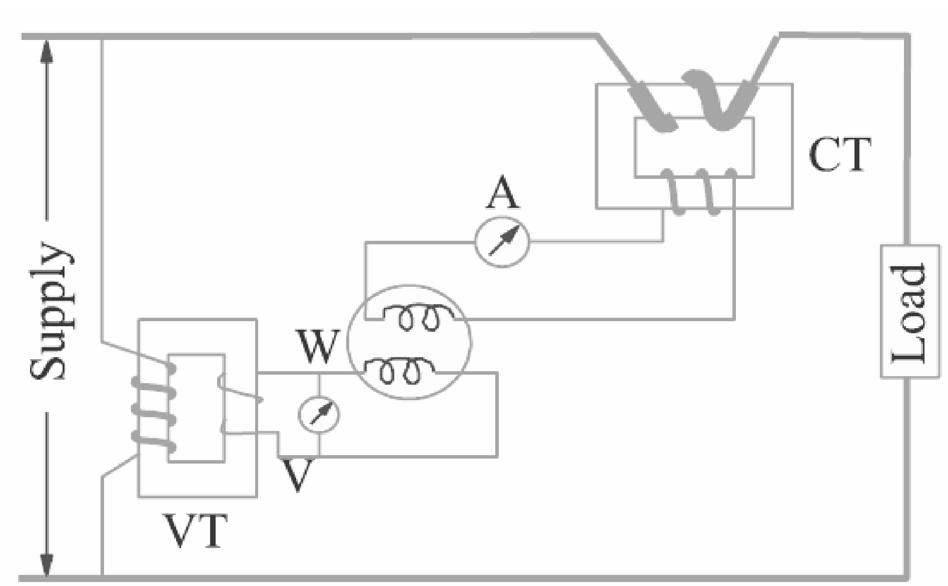


Figure: B

Terminals this is not the case with the ordinary constant-potential transformers because their primary current is determined by the load on their secondary whereas in a current transformer, primary current is determined entirely by the load on the system and not by the load on its own secondary. Hence, the secondary of a current transformer should never be left open under any circumstances.

10.71. Theory of Current Transformer:

Fig. (b) represents the general phase diagram for a current transformer. Current I_0 has been exaggerated for clarity.

(a) Ratio Error. For obtaining an expression for the ratio error, it will be assumed that the turn ratio n (= secondary turns, N2/primary turns N1) is made equal to the nominal current ratio i.e.

n = kn

In other words, it will be assumed that I_1/I_2 = n although actually n = I_1/I_2'

$$\sigma = \frac{n I_2 - I_1}{I_1} = \frac{I_2' - I_1}{I_1} = \frac{OB - OA}{OA} \qquad \text{— } [\because n = k_n]$$

$$\equiv \frac{OB - OC}{OA} \qquad (\because \beta \text{ is very small angle})$$

$$= -\frac{BC}{OA} = -\frac{AB \sin(\alpha + \delta)}{OA} = -\frac{I_0 \sin(\alpha + \delta)}{I_1} = -\frac{I_0 \sin(\alpha + \delta)}{nI_2}$$

For most instrument transformers, the power factor of the secondary burden is nearly unity so that δ is very small. Hence, very approximately

$$\sigma = \frac{I_0 \sin \alpha}{I_1} - \left| \frac{I_\omega}{I_1} \right.$$

Where $I\omega$ is the iron-loss or working or wattful component of the exciting current I_0

The transformation ratio *R* may be found from Fig. 10.85 (*a*) as under:

$I_1 = OA = OB + BC = nI_2 \cos\beta + I_0 \cos[90 - (\delta + \beta + \alpha)] = nI_2 \cos\beta + I_0 \sin(\delta + \beta + \alpha)$

Now $\beta = (\alpha + \delta)$ hence $I_1 = nI_2 + I_0 \sin(\alpha + \delta)$ where n is the turn ratio of the transformer.

$$\therefore \quad \text{ratio } R = \frac{I_1}{I_2} = \frac{nI_2 + I_0 \sin(\alpha + \delta)}{I_2} \text{ or } R = n + \frac{I_0 \sin(\alpha + \delta)}{I_2}$$

If δ is negligible small, then $R = n + \frac{I_0 \sin\alpha}{I_2} = n + \frac{I_\omega}{I_2}$

It is obvious from (*i*) above that ratio error can be eliminated if secondary turn are reduced by a number

$$= I_0 \sin(\alpha + \delta)/I_2$$

(b) Phase angle (β):

Again from Figure we fine that

$$\beta \cong \sin\beta = \frac{AC}{OA} = \frac{AB \cos(\alpha + \delta)}{OA} = \frac{I_0 \cos(\alpha + \delta)}{I_1} = \frac{I_0 \cos(\alpha + \delta)}{nI_2}$$

Again, if the secondary power factor is nearly unity, then δ is very small, hence

$$\beta \cong \frac{I_0 \cos\alpha}{I_1} = \frac{I_\mu}{I_1} \text{ or } \frac{I_\mu}{nI_2}$$

Where $I\mu$ is the magnetizing component of the exciting current I_0

$$\beta = \frac{I_\mu}{I_1} \quad \text{—in radian; } = \frac{180}{\pi} \times \frac{I_\mu}{I_1} \quad \text{—in degrees}$$

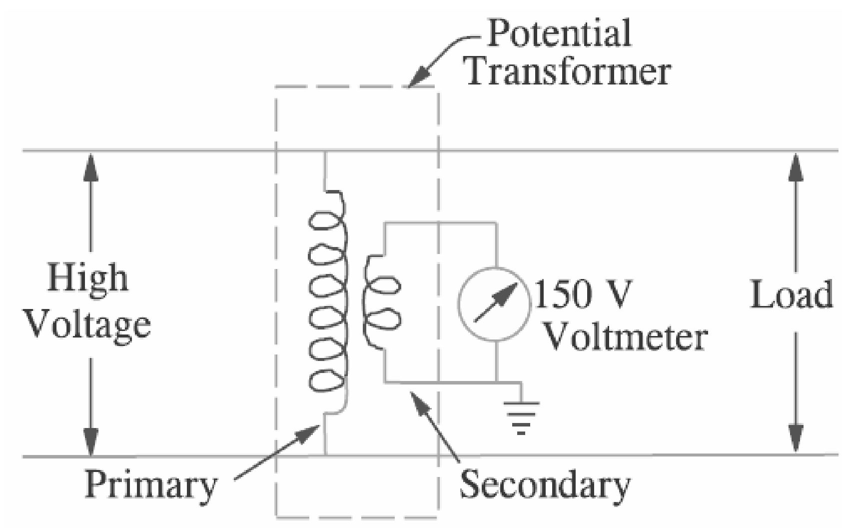
$$= \frac{I_\mu \cos\delta - I_\omega \sin\delta}{I_1} = \frac{I_\mu \cos\delta - I_\omega \sin\delta}{nI_2} \text{ radian}$$

$$\beta = \frac{180}{\pi} \times \frac{I_\mu \cos\delta - I_\omega \sin\delta}{nI_2} \text{ degrees.}$$

Dependence of ratio error on working component of I_0 and that of phase angle on the magnetizing component is obvious. If R is to come closer to k and β is to become negligible small, then $I\mu$ and $I\omega$ and hence I_0 should be very small.

Potential Transformers:

These transformers are extremely accurate-ratio step-down transformers and are used in conjunction with standard low-range voltmeters (100-120 V) whose deflection when divided by transformation ratio, gives the true voltage on the primary or high voltage side. In general, they are of the shell type and do not differ much from the ordinary two-winding transformers except that their power rating is extremely small. Sine their secondary windings are required to operate instruments or relays or pilot lights, their ratings are usually of 40 to 100W. For safety, the secondary is completely insulated from the high voltage primary and is, in addition, grounded for Affording protection to the operator figure shows the connection of such a transformer



Ratio and Phase-angle Errors:

In the case of a potential transformer, we are interested in the ratio of the primary to the secondary terminal voltage and in the phase angle γ between the primary and reversed secondary terminal voltage $V_{2'}$

The general theory of voltage transformer is the same as for the power transformers except that, as the current in the secondary burden is very small, the total primary current I_1 is not much greater than $I_{0.}$

In the phasor diagram of Figure, vectors *AB*, *BC*, *CD* and *DE* represent small voltage drops due to resistances and reactances of the transformer winding (they have been exaggerated for the sake of clarity). Since the drops as well as the phase angle γ are small, the top portion of diagram can be drawn with negligible loss of accuracy as in Figure where $V_{2'}$ vector has been drawn parallel to the vector for V_1.

In these diagrams, $V_{2'}$ is the secondary terminal voltage as referred to primary assuming transformation without voltage drops. All actual voltage drops have been referred to the primary. Vector AB represents total resistive drop as referred to primary i.e. $I_{2'}R_{01.}$ Similarly, BC represents total reactive drop as referred to primary i.e. $I_{2'}X_{01}$.

In a voltage transformer, the relatively large no-load current produces appreciable resistive drops which have been represented by vectors CD and DE respectively. Their values are I_0R_1 and I_0X_1 respectively.

(*a*) Ratio Error:

In the following theory, n would be taken to represent the ratio of primary turns to secondary turns. Further, it would be assumed, as before, that n equals the nominal transformation ratio i.e.

$$n = kn$$

In other words, it would be assumed that $V_1/V_2 = n$, although, actually, $V_1/V_{2'} = n$.

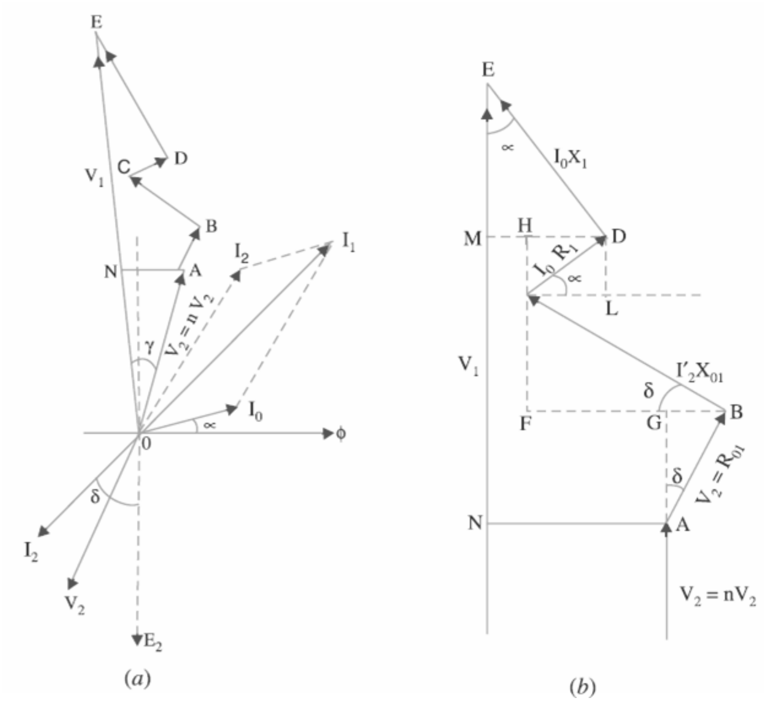
$$\sigma = \frac{k_n - k}{k} = \frac{k_n \cdot V_2 - k\,V_2}{kV_2} = \frac{V_2' - V_1}{V_1} = -\frac{EN}{OE}$$

$$= -\frac{AG + FC + LD + EM}{OE}$$

$$= -\frac{I_2'\,R_{02}\cos\delta + I_2'\,X_{02}\sin\delta + I_0 R_1 \sin\alpha + I_0 X_1 \cos\alpha}{V_1}$$

$$= -\frac{I_2'\,R_{02}\cos\delta + I_2'\,X_{02}\sin\delta + I_\mu R_1 + I_\mu X_1}{V_1}$$

Where $I\omega$ and $I\mu$ are the iron-loss and magnetizing components of the no-load primary current I_0



(*b*) Phase Angle (γ):

To a very close approximation, value of γ is given by γ = *AN/OA* —in radian

Now, $OA \cong OE$ provided ratio error is neglected. In that case,

$$\gamma = \frac{AN}{OE} = -\frac{GF + HM}{OE}$$

$$= -\frac{(BE - BG) + (DM - DH)}{OE}$$

$$= -\frac{I_2' X_{01} \cos\delta - I_2' R_{01} \sin\delta + I_0 X_1 \sin\alpha - I_0 R_1 \cos\alpha}{V_1}$$

$$= -\frac{I_2' X_{01} \cos\delta - I_2' R_{01} \sin\delta + I_\omega X_1 - I_\mu R_1}{V_1}$$

The negative sign has been given because reversed secondary voltage *i.e.* $V_{2'}$ lags behind V_1